References

[1] Carlson, H. W., "Correlation of sonic-boom theory with wind-tunnel and flight measurements," NASA TR R-213 (December 1964).

[2] Carlson, H. W., McLean, F. E., and Middleton, W. D., "Prediction of airplane sonic-boom pressure fields," NASA Conference on Aircraft Operating Problems, NASA SP-83, pp. 235–244(1965).

Chordwise Divergence of Rectangular Flat Plate Wings at Supersonic Speeds

A. J. Rhodes*

British Aircraft Corporation, Bristol, England

AND

D. J. Johns†

Loughborough College of Technology, Loughborough, England

Nomenclature

a	= speed of sound
A_E	= aeroelastic lift efficiency
C	= wing chord
C_{mn}, d_n	= generalized coordinates
E	= Young's modulus
$f_{(\vartheta)}$, $F_{(\vartheta)}$	= spanwise mode shapes
$g_{(\eta)}$, $G_{(\eta)}$	= chordwise mode shapes
l	= wing span (root to tip)
L_j	= lift/unit incidence of element j
m, n	= integers
M	= Mach number
r_m	= generalized coordinates
t	= wing thickness
W	= wing deflection
α	= incidence of rigid wing
α_j	= change in incidence, due to deformation, of element j
β	= $(M^2 - 1)^{1/2}$
η	= nondimensional chordwise coordinate
ζ	= nondimensional spanwise coordinate
ρ	= air density
$\varphi(\zeta,\eta)$	= deflection function

Introduction

In most of the published papers dealing with the chordwise divergence of rectangular flat plate wings clamped along their root chord it has been assumed that the spanwise mode shape is relatively unimportant, compared with the chordwise mode shape, and can be fixed arbitrarily. The wide variation in the results obtained (Fig. 1) has shown the need for analyses not embodying this assumption. Such analyses have been made by Rhodes[1] using completely general assumed deflection modes and Ackeret aerodynamic theory.

Previous studies using Ackeret aerodynamic theory have been made by Biot,[2,3] Broadbent[4] and Martin.[5] For a two-dimensional wing with no spanwise bending, Biot[2] derived an exact stability criterion for chordwise divergence which may be written

$$(E/\rho a^2)(t/l)^3 > 0.43(C/l)^3 M^2/\beta \tag{1}$$

The stabilizing influence of the anticlastic curvature induced by spanwise bending was shown in later papers by Biot,[2,3] and the influence of the assumed spanwise mode on the results was also indicated. Thus, if

$$W = f_{(\zeta)} \cdot g_{(\eta)} = \zeta^2(3 - \zeta) \sum_{n=0}^{6} d_n \eta^n \tag{2}$$

divergence was not possible for flat plates of aspect ratio 3 and 1.5.

More recent studies by the authors using as modes

$$\left.\begin{aligned} W &= \zeta^2(3 - \zeta) \sum_{n=0}^{2} d_n \eta^n + \zeta(2 - \zeta) \sum_{m=1}^{2} r_m \eta^m \\ \text{i. e.} \quad W &= f_{(\zeta)} \cdot g_{(\eta)} + F_{(\zeta)} G_{(\eta)} \end{aligned}\right\} \tag{3}$$

showed that divergence was possible at least for aspect ratios up to 2 (Fig. 1). Note that function $F \cdot G$ only satisfies the condition $\partial W/\partial \zeta = 0$ at $\eta = 0$.

Also shown in Fig. 1 are results using the modes

$$W = d_1 \zeta^3 \eta^2 + d_2 \zeta^2 \eta \tag{4}$$

as well as those from Refs. 1–5.

Broadbent[4] assumed that the plate deformed in a cylindrical mode about a generator through the root leading edge swept at 45° to the freestream. He obtained the following stability criterion:

$$(E/\rho a^2)(t/l)^3 > 0.3 M^2/\beta \tag{5}$$

Martin[5] assumed

$$W = (d_1 + d_2)\zeta^3\eta^2 + d_3[(\zeta^2/2) - (\zeta^3/3) + (\zeta^4/12)] + d_4\eta(2\zeta - \zeta^2) \tag{6}$$

where d_1 and d_2 are generalized coordinates for chordwise deformation forward of, and aft of, the mid chord, respectively. The results indicated that divergence was possible for low aspect ratio plate wings at high supersonic speeds.

Hancock[6] used an approximate, linearized, supersonic pressure distribution, which is much more general than Ackeret theory, and compared his results with Ref. 7, which used linearized slender wing theory and is only valid for low aspect ratios at $M \simeq 1$. Hancock showed, using as assumed modes

$$W = \tfrac{1}{26}(45\zeta^2 - 20\zeta^3 + \zeta^6)g_{(\eta)} \tag{7}$$

that chordwise divergence was not possible outside of the

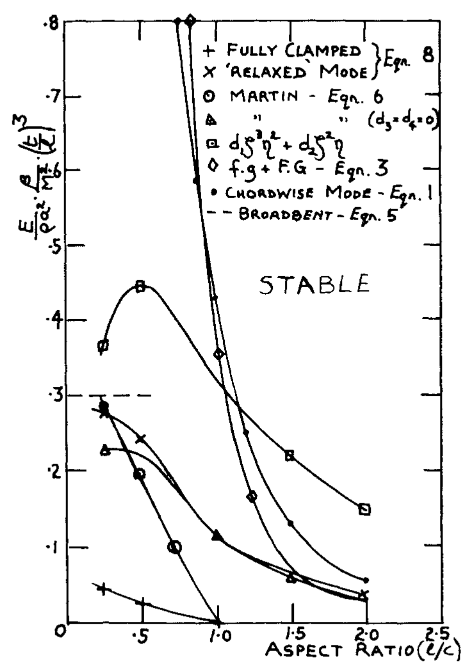

Fig. 1 Flat plate stability boundaries for $M \gg 1$.

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* Technical Officer.

† Reader in Aeronautical Engineering.

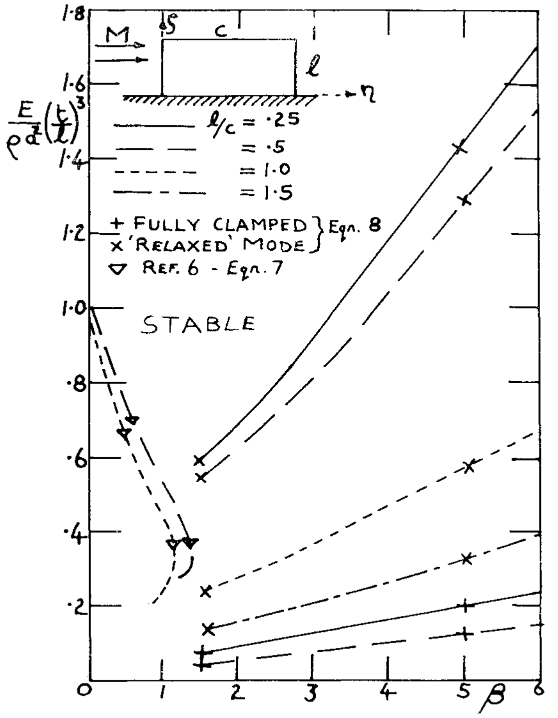

Fig. 2 Flat plate stability boundaries for $M \geq 1$ ($\beta \geq 0$).

range $1 < M < 1.8$ (Fig. 2). Also, differences in the stability criteria obtained in Refs. 6 and 7 for $M \simeq 1$ could be explained completely by the differences in the assumed spanwise mode shapes.

Solutions Using General Deflection Modes

Rhodes[1] assumed that

$$W = C\varphi_{(\zeta,\eta)} \tag{8}$$

where

$$\varphi_{(\zeta,\eta)} = \sum_{m=0}^{4} \sum_{n=0}^{4} C_{mn}\zeta^m\eta^n$$

The clamped root boundary conditions at $\zeta = 0$, for all η, imply that values of $m = 0, 1$ are excluded from Eq. (8). A stability determinant of order 15 is obtained, and the results for the stability criteria obtained are given in Figs. 1 and 2. Divergence was not possible at high supersonic speeds for aspect ratios greater than unity, and even for lower aspect ratios the critical plate thickness is so low as to virtually preclude divergence.

Results were also obtained for a relaxed mode in which the root boundary condition $\partial W/\partial\zeta = 0$ was only enforced at $\eta = 0$, the root leading edge. A stability determinant of order 19 was obtained, and the results are plotted in Figs. 1 and 2. The relaxed modes give larger areas of instability than the fully clamped modes and would suggest that divergence was possible at high supersonic speeds for aspect ratios at least up to 2.

Thus, as indicated by the results using Eqs. (2) and (3), relaxation of the spanwise mode shape through the root boundary conditions can indicate larger areas of instability than more exact analyses would give.

Solutions Using Structural Influence Coefficients

In Ref. 6 experimental influence coefficients have been obtained for a plate of aspect ratio 0.5 and used in a direct matrix analysis for divergence. The results confirm those obtained by energy analyses using assumed modes, Eq. (7), for values of $\beta = 0.45$ and 1.0 in Fig. 2.

Both experimental and theoretical structural influence coefficients have been used in Ref. 1, for plates of aspect ratio 2 and 1, respectively, with Ackeret aerodynamic theory.

The aeroclastic problem considered requires the solution of

$$\alpha_i = \sum_{j=1}^{N} L_j a_{ij}(\alpha + \alpha_j) \tag{9}$$

and the aeroclastic lift efficiency A_E (i.e., the ratio of lift on the elastic wing to that on the rigid wing)

$$A_E = \sum_{j=1}^{N} L_j(\alpha + \alpha_j) \Big/ \sum_{j=1}^{N} L_j\alpha \tag{10}$$

becomes, if $L_j = L = \text{const}$,

$$A_E = 1 + \frac{1}{N}\sum_{j=1}^{N}\frac{\alpha_j}{\alpha} \tag{11}$$

The calculations indicated that for aspect ratios 1 and 2 (with $N = 25$ and 50, respectively) the aeroclastic lift efficiency A_E is always less than unity and divergence (when $A_E \rightarrow \infty$) is not possible. These results confirm those obtained with the fully clamped mode in the energy analyses described earlier.

Conclusions

The foregoing studies confirm that divergence is most unlikely at high supersonic speeds, for rectangular flat plate wings, on the basis of linear structural theory and Ackeret aerodynamic theory. Also, the choice of assumed modes in divergence analyses is very critical and completely general spanwise and chordwise mode shapes should be considered which satisfy all of the root boundary conditions.

References

[1] Rhodes, A. J., "The chordwise divergence of rectangular flat plate wings at supersonic speeds," College of Aeronautics, Cranfield, England (June 1962); unpublished.

[2] Biot, M. A., "Aeroelastic stability of supersonic wings," Repts. 1–6, Cornell Aeronautical Lab. (December 1947–March 1952).

[3] Biot, M. A., "The divergence of supersonic wings including chordwise bending," Rept. 67, Cornell Aeronautical Lab. (December 1954).

[4] Broadbent, E. G., "A simple criterion for the divergence of rectangular thin flat plate wings of low aspect ratio at supersonic speeds," Royal Aircraft Establishment (January 1949); unpublished.

[5] Martin, E. C., "Wing divergence at supersonic speeds," College of Aeronautics, Cranfield, England (May 1955); unpublished.

[6] Hancock, G. J., "Divergence of plate aerofoils of low aspect ratio at supersonic speeds," J. Aerospace Sci. 26, 495–507, 517 (1959).

[7] Hedgepeth, J. M. and Waner, P. G., "Analysis of static aeroelastic behaviour of low aspect ratio rectangular wings," NACA TN 3958 (April 1957).